A four–neutrino texture implying bimaximal flavor mixing and reduced LSND effect*

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Abstract

A four–neutrino effective texture is described, where a sterile neutrino mixes nearly maximally with the electron neutrino and so, is responsible for the deficit of solar ν_e's (according to the large–angle MSW solution or vacuum solution, of which the former is selected *a posteriori*). But, while maximal mixing of muon neutrino with tauon neutrino causes the deficit of atmospheric ν_μ's, the original magnitude of LSND effect is reduced by as much as four orders, becoming unobservable.

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As is well known, in addition to three active neutrinos ν_e, ν_μ, ν_τ, one sterile neutrino ν_s, at least, is needed to explain in terms of neutrino oscillations three neutrino effects: the deficits of solar ν_e's and atmospheric ν_μ's as well as the possible LSND excess of ν_e's in accelerator beam of ν_μ's [1]. This is a phenomenological reason for introducing sterile neutrinos. From the theoretical viewpoint, however, sterile neutrinos may exist in Nature, whether the LSND effect is real or not.

In this paper, we describe a four–neutrino effective texture implying bimaximal mixing of ν_e with ν_s and ν_μ with ν_τ, but, at the same time, only a tiny LSND effect, reduced by as much as four orders of magnitude in comparison with its original estimation.

In our texture, the mass matrix for active neutrinos ν_e, ν_μ, ν_τ gets the same form $M = (M_{\alpha\beta})$ $(\alpha,\, \beta = e,\, \mu,\, \tau)$ as the mass matrix for charged leptons e^-, μ^-, τ^- (only the values of parameters are expected to be different). In order to operate with an explicit model, we accept in both cases the ansatz [2]

$$(M_{\alpha\beta}) = \frac{1}{29}\left(\begin{array}{ccc} \mu\varepsilon & 2\alpha & 0 \\ 2\alpha & 4\mu(80+\varepsilon)/9 & 8\sqrt{3}\,\alpha \\ 0 & 8\sqrt{3}\,\alpha & 24\mu(624+\varepsilon)/25 \end{array}\right) , \tag{1}$$

where $\varepsilon > 0$, $\mu > 0$ and $\alpha > 0$ are three parameters, taking different values for neutrinos and charged leptons.

In the case of charged leptons, the ansatz (1) leads for $\alpha \to 0$ to the prediction

$$m_\tau \to 1776.80 \;\mathrm{MeV}\,,\; \varepsilon \to 0.172329\,,\; \mu \to 85.9924 \;\mathrm{MeV}\,, \tag{2}$$

if experimental values of m_e and m_μ are used as an input. In fact, the lowest perturbative calculation with respect to α/μ, when applied to the eigenvalue equation for the matrix (1), gives in particular [2]

$$\begin{aligned} m_\tau &= \frac{6}{125}\,(351 m_\mu - 136 m_e) + 10.2112\left(\frac{\alpha}{\mu}\right)^2 \;\mathrm{MeV} \\ &= \left[1776.80 + 10.2112\left(\frac{\alpha}{\mu}\right)^2\right] \;\mathrm{MeV}\,. \end{aligned} \tag{3}$$

When the experimental value $m_\tau^{\mathrm{exp}} = 1777.05^{+0.29}_{-0.26}$ [3] is used, Eq. (3) implies

$$\left(\frac{\alpha}{\mu}\right)^2 = 0.024^{+0.028}_{-0.025} \,, \tag{4}$$

what is not inconsistent with $\alpha = 0$ (then M becomes diagonal). Impressive agreement of the prediction for m_τ with the experimental $m_\tau^{\rm exp}$ is our phenomenological motivation for the use of form (1) as the lepton mass matrix M. Methodologically, we consider here our form (1) of M as a detailed ansatz, though it can be somehow theoretically supported (the interested reader may find some arguments in Appendix to Ref. [4]).

In contrast to the charged–lepton case, where $\alpha/\mu \ll 1$ (and so, M is nearly diagonal), we conjecture in the neutrino case that $\mu/\alpha \ll 1$ (and it is small enough to get M nearly off–diagonal). The reason is that only in such a situation we can expect nearly maximal neutrino mixing, namely of ν_μ with ν_τ as it is preferably suggested by Super–Kamiokande experiments on the deficit of atmospheric ν_μ's [5]. Then, in order to explain potentially also the deficit of solar ν_e's [6] as well as the possible LSND effect for accelerator ν_μ's [7], we accept the popular hypothesis [1] that in Nature there is a sterile neutrino ν_s which may mix with active neutrinos ν_e, ν_μ, ν_τ, dominantly with ν_e.

To construct an effective model of four–neutrino texture, we assume that the mass matrix for neutrinos ν_s, ν_e, ν_μ, ν_τ has the 4×4 form $M = (M_{\alpha\beta})$ $(\alpha,\, \beta = s,\, e,\, \mu,\, \tau)$, where

$$M_{s\,s} = 0 \;,\; M_{s\,e} = \lambda M_{e\,\mu} = M_{e\,s} \;,\; M_{s\,\mu} = 0 = M_{\mu\,s} \;,\; M_{s\,\tau} = 0 = M_{\tau\,s} \tag{5}$$

are seven new matrix elements, while the rest of them are old, as given in Eq. (1). Here, the ratio $\lambda \equiv M_{s\,e}/M_{e\,\mu} > 0$ is a neutrino fourth free parameter. The old neutrino free parameter ε will be put zero (as seen from Eq. (2), even for charged leptons ε is small). Then,

$$M_{e\,e} = 0 \;,\; M_{\mu\,\mu} = \frac{4}{9}\, 80\frac{\mu}{29} \;,\; M_{\tau\,\tau} = \frac{24}{25}\, 624\frac{\mu}{29} \;. \tag{6}$$

The ratios

$$\xi \equiv \frac{M_{\tau\,\tau}}{M_{e\,\mu}} = 299.52\frac{\mu}{\alpha} \quad , \quad \chi \equiv \frac{M_{\mu\,\mu}}{M_{e\,\mu}} = \frac{1}{16.848}\xi \tag{7}$$

are small, when $\mu/\alpha \ll 1$ is small enough.

Now, solving the eigenvalue equation for the 4×4 matrix M in the first perturbative order with respect to ξ, we obtain the following neutrino masses:

$$
\begin{aligned}
m_0 &= \frac{2\alpha}{29}\left\{-\frac{1}{\sqrt{2}}\left[49+\lambda^2-\sqrt{(49-\lambda^2)^2+4\lambda^2}\right]^{1/2}+\frac{1}{2}\frac{1}{49}\xi\right\} \simeq \frac{2\alpha}{29}\left[-\sqrt{\frac{48}{49}}\lambda+\frac{1}{2}\frac{1}{49}\xi\right] ,\\
m_1 &= \frac{2\alpha}{29}\left\{\;\frac{1}{\sqrt{2}}\left[49+\lambda^2-\sqrt{(49-\lambda^2)^2+4\lambda^2}\right]^{1/2}+\frac{1}{2}\frac{1}{49}\xi\right\} \simeq \frac{2\alpha}{29}\left[\sqrt{\frac{48}{49}}\lambda+\frac{1}{2}\frac{1}{49}\xi\right] ,\\
m_2 &= \frac{2\alpha}{29}\left\{-\frac{1}{\sqrt{2}}\left[49+\lambda^2+\sqrt{(49-\lambda^2)^2+4\lambda^2}\right]^{1/2}+\frac{1}{2}\left(\frac{48}{49}\xi+\chi\right)\right\}\\
&\simeq \frac{2\alpha}{29}\left[-7+\frac{1}{2}\left(\frac{48}{49}\xi+\chi\right)\right] ,\\
m_3 &= \frac{2\alpha}{29}\left\{\;\frac{1}{\sqrt{2}}\left[49+\lambda^2+\sqrt{(49-\lambda^2)^2+4\lambda^2}\right]^{1/2}+\frac{1}{2}\left(\frac{48}{49}\xi+\chi\right)\right\}\\
&\simeq \frac{2\alpha}{29}\left[7+\frac{1}{2}\left(\frac{48}{49}\xi+\chi\right)\right] .
\end{aligned}
\tag{8}
$$

Here, the second step is valid in the linear approximation in λ, what requires small $\lambda/49$, while the former perturbative calculation with respect to ξ works for small $\xi/7$. We can conclude from Eqs. (8) that $m_3 \gtrsim |m_2| \gg m_1 \gtrsim |m_0|$.

The neutrino diagonalizing 4×4 matrix $U=(U_{\alpha i})$ $(\alpha = s,\, e,\, \mu,\, \tau,\;\; i=0,\,1,\,2,\,3)$, such that $U^\dagger M U = \mathrm{diag}\,(m_0,\, m_1,\, m_2,\, m_3)$, gets in the zero order with respect to ξ and in the linear approximation in λ the following form:

$$
(U_{\alpha i}) \simeq \left(\begin{array}{cccc}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\lambda}{49\sqrt{2}} & \frac{\lambda}{49\sqrt{2}} \\
-\frac{\sqrt{48}}{7\sqrt{2}} & \frac{\sqrt{48}}{7\sqrt{2}} & -\frac{1}{7\sqrt{2}} & \frac{1}{7\sqrt{2}} \\
-\frac{\lambda}{49\sqrt{2}} & -\frac{\lambda}{49\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{7\sqrt{2}} & -\frac{1}{7\sqrt{2}} & -\frac{\sqrt{48}}{7\sqrt{2}} & \frac{\sqrt{48}}{7\sqrt{2}}
\end{array}\right) + O(\xi/7) \,. \tag{9}
$$

Evidently, in this case $\xi/7$ ought to be smaller than $\lambda/49$. If the charged–lepton diagonalizing 3×3 matrix is nearly unit due to the small value (4) of α/μ, the lepton counterpart $V=(V_{i\,\alpha})$ of the quark Cabibbo—Kobayashi—Maskawa matrix is approximately equal to $U^\dagger = \left(U^\dagger{}_{i\,\alpha}\right) = (U^*_{\alpha\, i})$. Thus, in this approximation, the fields

$$
\nu_i = \sum_\alpha V_{i\,\alpha}\nu_\alpha = \sum_\alpha U^*_{\alpha\, i}\nu_\alpha \tag{10}
$$

describe four massive neutrinos ν_i $(i=0,1,2,3)$ in terms of four flavor neutrinos ν_α $(\alpha = s,\, e,\, \mu,\, \tau)$. Hence,

$$\nu_\alpha = \sum_i U_{\alpha\, i} \nu_i \;\;,\;\; |\nu_\alpha\rangle = \nu_\alpha^\dagger |0\rangle = \sum_i U^*_{\alpha\, i} |\nu_i\rangle \;. \tag{11}$$

Then, the neutrino oscillation probabilities on the energy shell E read

$$\begin{aligned} P\left(\nu_\alpha \to \nu_\beta\right) &= |\langle \nu_\beta | e^{iPL} | \nu_\alpha\rangle|^2 \\ &= \delta_{\beta\,\alpha} - 4 \sum_{j>i} U^*_{\beta\, j} U_{\alpha\, j} U_{\beta\, i} U^*_{\alpha\, i} \sin^2 x_{ji} \;, \end{aligned} \tag{12}$$

where L denotes the experimental baseline and

$$x_{ji} = 1.27 \frac{\Delta m^2_{ji} L}{E} \;\;,\;\; \Delta m^2_{ji} = m^2_j - m^2_i \tag{13}$$

with Δm_{ji}, L and E expressed in eV, km and GeV, respectively. In Eq. (12) the eigenvalues of momentum operator P are $p_i = \sqrt{E^2 - m_i^2} \simeq E - m_i^2/2E$. Evidently, because of real $M_{\alpha\,\beta}$ and thus real $U_{\alpha\, i}$, the possible CP violation is here neglected.

From Eqs. (12) and (9) we calculate in the zero perturbative order with respect to ξ and linear approximation in λ the following oscillation probabilities:

$$\begin{aligned} P\left(\nu_e \to \nu_e\right) &\simeq 1 - \frac{48^2}{49^2} \sin^2 x_{10} - \frac{4 \cdot 48}{49^2} \sin^2 x_{21} - \frac{1}{49^2} \sin^2 x_{32} \;, \\ P\left(\nu_\mu \to \nu_\mu\right) &\simeq 1 - \sin^2 x_{32} \;, \\ P\left(\nu_\mu \to \nu_e\right) &\simeq \frac{1}{49} \sin^2 x_{32} \;. \end{aligned} \tag{14}$$

In the first and third formula (14) we put approximately $\Delta m^2_{20} \simeq \Delta m^2_{30} \simeq \Delta m^2_{21} \simeq \Delta m^2_{31}$ due to Eqs. (8) with $\xi \simeq 0$ (then, a linear term in λ appearing in the third formula vanishes). Note from Eqs. (8) that

$$\Delta m^2_{10} \simeq \frac{2}{49} \sqrt{\frac{48}{49}} \left(\frac{2\alpha}{29}\right)^2 \lambda\, \xi \;\;,\;\; \Delta m^2_{32} \simeq 14 \left(\frac{2\alpha}{49}\right)^2 \left(\frac{48}{49}\xi + \chi\right) \tag{15}$$

for small $\lambda/49$ and $\xi/7$. Here, $\chi = 5.9354 \times 10^{-2} \xi$.

If $1.27\, \Delta m^2_{32}\, L_{\rm atm}/E_{\rm atm} = O(1)$ and $\Delta m^2_{32} \leftrightarrow \Delta m^2_{\rm atm} \sim 3 \times 10^{-3}$ eV^2 [5], the second formula (14) is able to describe oscillations of atmospheric ν_μ's (dominantly into ν_τ's) with maximal amplitude. Then, the second Eq. (15) gives the estimate

$$\alpha^2 \xi \sim 4.3 \times 10^{-2}\ \mathrm{eV}^2\ . \tag{16}$$

Hence, if one assumes reasonably that $\alpha \leq O(1\ \mathrm{eV})$, one gets $\xi \geq O(10^{-2})$.

On the other hand, if $1.27\, \Delta m^2_{10}\, L_{\rm sol}/E_{\rm sol} = O(1)$ and $\Delta m^2_{10} \leftrightarrow \Delta m^2_{\rm sol} \sim 10^{-5}\ \mathrm{eV}^2$ or $10^{-10}\ \mathrm{eV}^2$ [6], the first formula (14) can describe respectively large–angle MSW oscillations or vacuum oscillations of solar ν_e's (dominantly into ν_s's) with nearly maximal amplitude. In fact, it implies

$$P\left(\nu_e \rightarrow \nu_e\right) \simeq 1 - \frac{48^2}{49^2} \sin^2 x_{10} - \frac{4 \cdot 48 + 1}{2 \cdot 49^2} \simeq 1 - \frac{48^2}{49^2} \sin^2 x_{10} \tag{17}$$

due to $x_{10} \ll x_{32} \ll x_{21}$. Then, from the first Eq. (15) we get the estimate

$$\alpha^2 \lambda\, \xi \sim 5.2 \times 10^{-2}\ \mathrm{eV}^2 \ \text{ or } \ 5.2 \times 10^{-7}\ \mathrm{eV}^2\ , \tag{18}$$

respectively.

Thus, we find from Eqs. (16) and (18) that

$$\lambda \sim 1.2 \ \text{ or } \ 1.2 \times 10^{-5}\ , \tag{19}$$

respectively. This shows that the matrix element $M_{s\,e}$ is comparable or small *versus* $M_{e\,\mu}$. Evidently, only the first option (related to large–angle MSW oscillations of solar ν_e's) can be compatible with the mixing matrix (9) and so, with the oscillation formulae (14) leading to the nearly maximal mixing of ν_s with ν_e. In fact, only in this option, ξ may be smaller than λ, as required by the form (9) of neutrino mixing matrix.

In the case of Chooz experiment searching for oscillations of reactor $\bar{\nu}_e$'s [8], where it happens that $1.27\, \Delta m^2_{32}\, L_{\rm Chooz}/E_{\rm Chooz} = O(1)$, the first formula (14) leads to

$$P\left(\bar{\nu}_e \rightarrow \bar{\nu}_e\right) \simeq 1 - \frac{1}{49^2} \sin^2 x_{32} - \frac{2 \cdot 48}{49^2} \simeq 1 \tag{20}$$

since $x_{10} \ll x_{32} \ll x_{21}$. This is consistent with the negative result of Chooz experiment.

The third formula (14) implies te existence of $\nu_\mu \rightarrow \nu_e$ neutrino oscillations with the amplitude equal to $1/49 \simeq 0.02$ and the mass–squared scale given by Δm^2_{32}. Such an amplitude is compatible with the LSND estimation, say, $\sin^2 2\theta_{\rm LSND} \sim 0.02$, but the

mass–squared scale Δm^2_{32} — being equal to the atmospheric $\Delta m^2_{\rm atm} \sim 3 \times 10^{-3}$ eV^2 — is smaller than the LSND estimation, say, $\Delta m^2_{\rm LSND} \sim 0.5$ eV^2 [7] roughly by two orders of magnitude.

In conclusion, our four–neutrino effective texture may describe correctly both deficits of solar ν_e's and atmospheric ν_μ's. Then, it predicts the existence of a tiny LSND effect of the magnitude reduced by four orders in comparison with the original LSND estimation. It is so, because

$$\sin^2\left(1.27\frac{\Delta m^2_{32}\, L_{\rm LSND}}{E_{\rm LSND}}\right) \sim \sin^2\left(1.27\frac{10^{-2}\Delta m^2_{\rm LSND}\, L_{\rm LSND}}{E_{\rm LSND}}\right) \sim 10^{-4} \tag{21}$$

for $1.27\,\Delta m^2_{32}\, L_{\rm LSND}/E_{\rm LSND} \sim 1$. This reduced LSND effect would be, therefore, practically unobservable (for original $L = L_{\rm LSND}$ and $E = E_{\rm LSND}$).

Obviously, the experimental problem of existence of the LSND effect, or of another realization of $\nu_\mu \rightarrow \nu_e$ neutrino oscillations, is crucial for all discussions about neutrino texture. In particular, a clear confirmation of the original LSND effect would exclude our four–neutrino effective texture.

In such a case, the option of three pseudo–Dirac neutrinos might be invoked to explain all three neutrino–oscillation effects: the deficits of solar ν_e's and atmospheric ν_μ's as well as the LSND effect (*cf. e.g.*, Refs. [4] and [9]). This option involves three natural Majorana sterile neutrinos mixing nearly maximally with three Majorana active neutrinos, and produces three pairs of light mass–neutrino states. It is in contrast to the popular see–saw option, where the natural Majorana sterile neutrinos and Majorana active neutrinos practically do not mix, and where they produce heavy and light mass–neutrino states, respectively. In the see–saw option, small masses of the latter states are conditioned by large masses of the former.

References

1. *Cf. e.g.*, C. Giunti, Talk at the ICFA/ECFA Workshop, Lyon, July 1999, hep–ph/9910336; and references therein.
2. W. Królikowski, *Acta Phys. Pol.* **B 30**, 2631 (1999); and references therein.
3. Review of Particle Physics, *Eur. Phys. J.* **C 3**, 1 (1998).
4. W. Królikowski, "Oscillations of the mixed pseudo–Dirac neutrinos", *Nuovo Cim.* **A** (to appear); and references therein.
5. Y. Fukuda *et al.* (Super–Kamiokande Collaboration), *Phys. Rev. Lett.* **81**, 1562 (1998).
6. *Cf. e.g.*, J.N. Bahcall, P.I. Krastev and A.Y. Smirnov, *Phys. Rev.* **D 58**, 096016 (1998); hep–ph/9905220v2.
7. C. Athanassopoulos *et al.* (LSND Collaboration), *Phys. Rev. Lett.* **75**, 2650 (1995); *Phys. Rev.* **C 54**, 2685 (1996); *Phys. Rev. Lett.* **77**, 3082 (1996); **81**, 1774 (1998).
8. M. Appolonio *et al.* (Chooz Collaboration), *Phys. Lett.* **B 420**, 397 (1998).
9. W. Królikowski, hep–ph/9910308 (to appear in *Acta Phys. Pol.* **B**); and references therein.